

Quantum Optics with Atomic Motion

Christopher Monroe



UNIVERSITY OF
MARYLAND



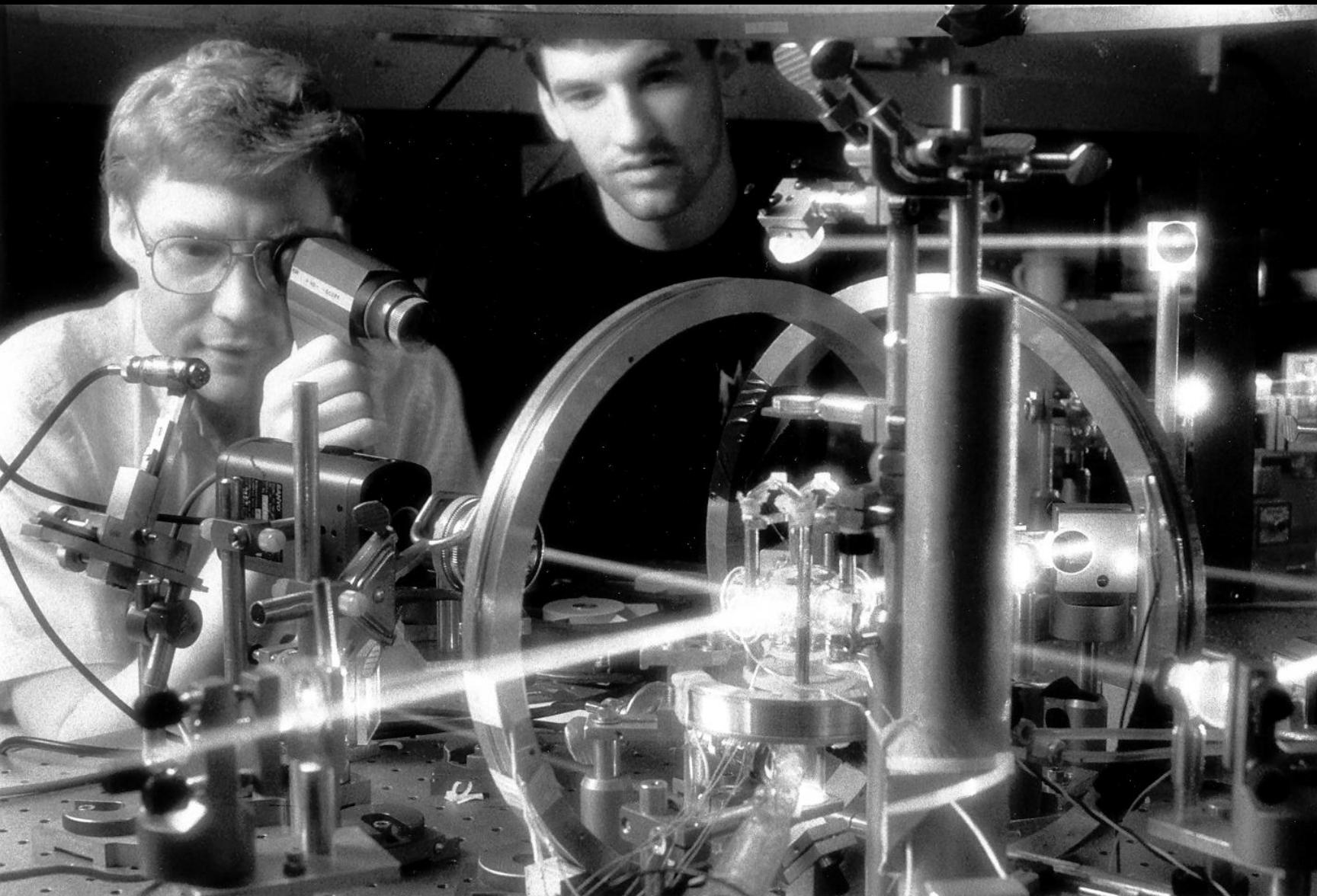
The Quantum Theory of Optical Coherence*

Roy J. GLAUBER

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

(Received 11 February 1963)

The concept of coherence which has conventionally been used in optics is found to be inadequate to the needs of recently opened areas of experiment. To provide a fuller discussion of coherence, a succession of correlation functions for the complex field strengths is defined. The n th order function expresses the correlation of values of the fields at $2n$ different points of space and time. Certain values of these functions are measurable by means of n -fold delayed coincidence detection of photons. A fully coherent field is defined as one whose correlation functions satisfy an infinite succession of stated conditions. Various orders of incomplete coherence are distinguished, according to the number of coherence conditions actually satisfied. It is noted that the fields historically described as coherent in optics have only first-order coherence. On the other hand, the existence, in principle, of fields coherent to all orders is shown both in quantum theory and classical theory. The methods used in these discussions apply to fields of arbitrary time dependence. It is shown, as a result, that coherence does not require monochromaticity. Coherent fields can be generated with arbitrary spectra.



Coherent and Incoherent States of the Radiation Field*

ROY J. GLAUBER

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

(Received 29 April 1963)

Methods are developed for discussing the photon statistics of arbitrary radiation fields in fully quantum-mechanical terms. In order to keep the classical limit of quantum electrodynamics plainly in view, extensive use is made of the coherent states of the field. These states, which reduce the field correlation functions to factorized forms, are shown to offer a convenient basis for the description of fields of all types. Although they are not orthogonal to one another, the coherent states form a complete set. It is shown that any quantum state of the field may be expanded in terms of them in a unique way. Expansions are also developed

$$\mathbf{E}^{(+)}(\mathbf{r}t) = i \sum_k (\frac{1}{2}\hbar\omega_k)^{1/2} a_k \mathbf{u}_k(\mathbf{r}) e^{-i\omega_k t}. \quad (2.19)$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^n}{(n!)^{1/2}} |n\rangle \quad (3.7)$$

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}. \quad (3.17)$$

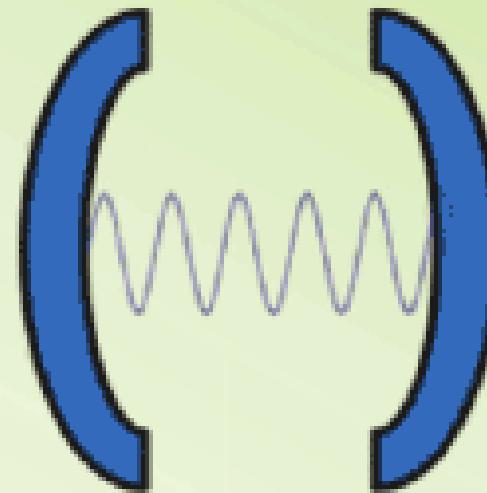
Control of Individual Quantum Systems

(see Nobel Prize 2012, Wineland and Haroche)

Ion in a trap



Photon in a cavity



$$Q = 4 \times 10^{10}$$
$$T_c = 130 \text{ ms}$$



Spin-motion coupling: 2LS+QHO

$$H = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \underbrace{\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2}_{\hbar\omega(a^+a+1/2)} - \hat{\mu} \cdot E(\hat{x})$$

frequency of applied radiation

$$\text{Rabi frequency } \hbar g$$

$$-\mu_0 \cdot \frac{E_0}{2} (\hat{\sigma}_+ + \hat{\sigma}_-) (e^{ik\hat{x}-i\omega_L t} + e^{-ik\hat{x}+i\omega_L t})$$

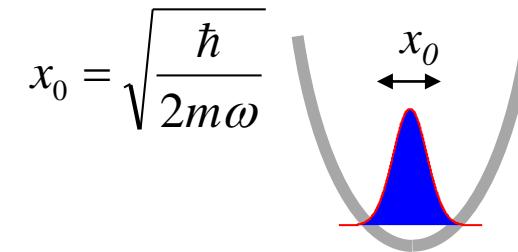
interaction frame, “rotating wave approximation”

$$H = \hbar g(\hat{\sigma}_+ e^{ik\hat{x}-i\delta t} + \hat{\sigma}_- e^{-ik\hat{x}+i\delta t})$$

$$\hat{x} = x_0(a e^{-i\omega t} + a^+ e^{i\omega t})$$

$$\delta = \omega_L - \omega_0 = \text{detuning}$$

$$k = 2\pi/\lambda = \text{wavenumber}$$



$$H = \hbar g [\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t} + a^+ e^{i\omega t}) - i\delta t} + \hat{\sigma}_- e^{-ikx_0(ae^{-i\omega t} + a^+ e^{i\omega t}) + i\delta t}]$$

$\eta = kx_0$ = "Lamb-Dicke parameter" ~ 0.1

Stationary terms arise in H at particular values of δ :

(1) $\delta = 0$

$$\begin{aligned} H_0 &= \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-) \left\{ 1 - \frac{\eta^2}{2!}(a^\dagger a + a a^\dagger) \right. \\ &\quad + \frac{\eta^4}{4!}(a^\dagger a^\dagger a a + a^\dagger a a a^\dagger + a a^\dagger a a^\dagger + a a^\dagger a^\dagger a) \\ &\quad \left. - \eta^6(a^\dagger a^\dagger a^\dagger a a a + \dots) \right\} \end{aligned}$$

$$\approx \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)(1 - n\eta^2) \approx \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)$$

$kx_0\sqrt{n+1} \ll 1$
"Lamb-Dicke limit"

"Carrier": $\langle \downarrow, n | H_0 | \uparrow, n \rangle = \hbar g$

$$H = \hbar g [\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t} + a^+ e^{i\omega t}) - i\delta t} + \hat{\sigma}_- e^{-ikx_0(ae^{-i\omega t} + a^+ e^{i\omega t}) + i\delta t}]$$

$\eta = kx_0$ = “Lamb-Dicke parameter”

Stationary terms arise in H at particular values of δ :

(2) $\delta = -\omega$

$$H_- = \hbar g \hat{\sigma}_+ \left\{ \eta a - \frac{\eta^3}{3!} (a^\dagger a a + a a^\dagger a + a a a^\dagger) + \dots \right\} + h.c.$$

$$\approx \hbar g (\hat{\sigma}_+ a + \hat{\sigma}_- a^\dagger)$$

"Red Sideband": $\langle \uparrow, n-1 | H_0 | \downarrow, n \rangle = \hbar g \sqrt{n}$

$kx_0 \sqrt{n+1} \ll 1$
“Lamb-Dicke limit”

$$H = \hbar g [\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t} + a^+ e^{i\omega t}) - i\delta t} + \hat{\sigma}_- e^{-ikx_0(ae^{-i\omega t} + a^+ e^{i\omega t}) + i\delta t}]$$

$\eta = kx_0$ = “Lamb-Dicke parameter”

Stationary terms arise in H at particular values of d :

(3) $\delta = \omega$

$$H_+ = \hbar g \hat{\sigma}_+ \left\{ \eta a^\dagger - \frac{\eta^3}{3!} (a^\dagger a^\dagger a + a a^\dagger a^\dagger + a^\dagger a a^\dagger) + \dots \right\} + h.c.$$

$$\approx \hbar g (\hat{\sigma}_+ a^\dagger + \hat{\sigma}_- a)$$

“Blue Sideband”: $\langle \uparrow, n+1 | H_+ | \downarrow, n \rangle = \hbar g \sqrt{n+1}$

$kx_0 \sqrt{n+1} \ll 1$
 “Lamb-Dicke limit”

$$\langle \uparrow, n' | H | \downarrow, n \rangle = \hbar g e^{-\eta^2/2} \eta^{|n'-n|} \sqrt{\frac{n_{<}!}{n_{>}!}} L_{n_{<}}^{|n'-n|}(\eta^2)$$

Ordered Expansions in Boson Amplitude Operators*

K. E. CAHILL† AND R. J. GLAUBER

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 8 July 1968)

The expansion of operators as ordered power series in the annihilation and creation operators a and a^\dagger is examined. It is found that normally ordered power series exist and converge quite generally, but that for the case of antinormal ordering the required c -number coefficients are infinite for important classes of operators. A parametric ordering convention is introduced according to which normal, symmetric, and antinormal ordering correspond to the values $s = +1, 0, -1$, respectively, of an order parameter s . In

We shall now express the matrix elements of the displacement operator $D(\alpha)$ in the n -quantum representation in terms of the associated Laguerre polynomials $L_n^{(m)}(x)$. We first note that if $|\alpha\rangle$ is a coherent state then, by using Eqs. (2.19) and (2.20), we may write

$$\begin{aligned} \langle m | D(\xi) | \alpha \rangle &= D(\xi) D(\alpha) | 0 \rangle \\ &= D(\xi + \alpha) | 0 \rangle \exp\left[\frac{1}{2}(\xi\alpha^* - \xi^*\alpha)\right] \\ &= |\xi + \alpha\rangle \exp\left[\frac{1}{2}(\xi\alpha^* - \xi^*\alpha)\right]. \end{aligned}$$

Thus, by using Eq. (2.23), we find that

$$\begin{aligned} \langle m | D(\xi) | \alpha \rangle &= (m!)^{-1/2} (\xi + \alpha)^m \exp\left[\frac{1}{2}(\xi\alpha^* - \xi^*\alpha)\right] \\ &\quad - \frac{1}{2} |\xi + \alpha|^2 \\ &= (m!)^{-1/2} (\xi + \alpha)^m \exp\left(-\frac{1}{2} |\xi|^2 - \frac{1}{2} |\alpha|^2 - \xi^*\alpha\right). \end{aligned} \quad (B1)$$

Another expression for this same matrix element also follows from Eq. (2.23):

$$\langle m | D(\xi) | \alpha \rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^{1/2}} \langle m | D(\xi) | n \rangle. \quad (B2)$$

If we now combine Eqs. (B1) and (B2) and put $y = \xi^{-1}\alpha$, we arrive at the relation

$$\begin{aligned} (1+y)^m e^{-y|\xi|^2} &= e^{-|\xi|^2/2} \sum_{n=0}^{\infty} \left(\frac{m}{n!}\right)^{1/2} \xi^{n-m} \\ &\times \langle m | D(\xi) | n \rangle y^n. \end{aligned} \quad (B3)$$

The left-hand side of this equation is a generating function for the associated Laguerre polynomials $L_n^{(m)}(x)$ according to the identity¹⁹

$$(1+y)^m e^{-xy} = \sum_{n=0}^{\infty} L_n^{(m-n)}(x) y^n, \quad (B4)$$

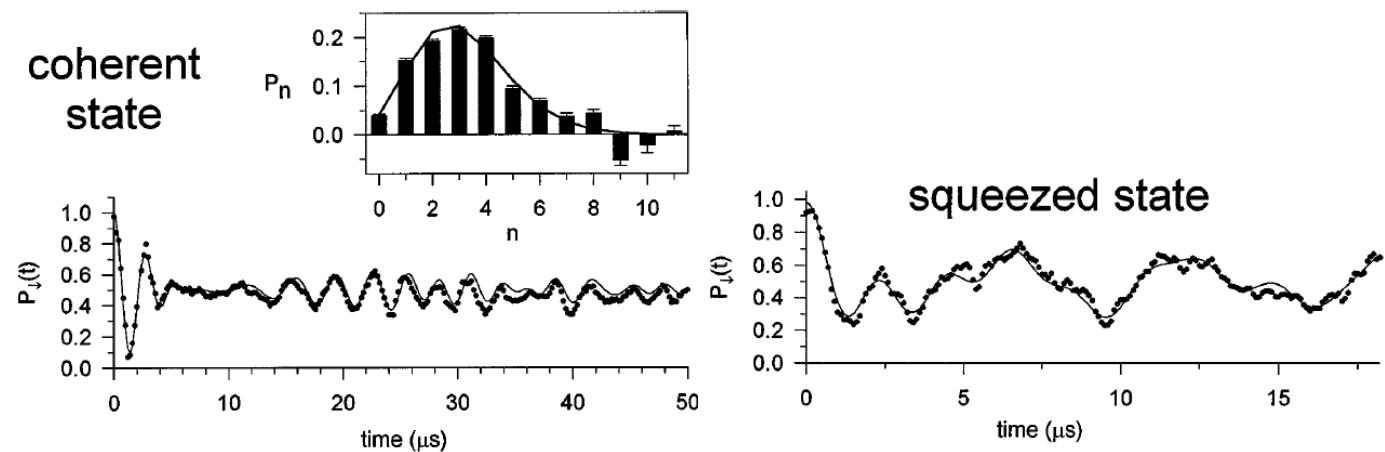
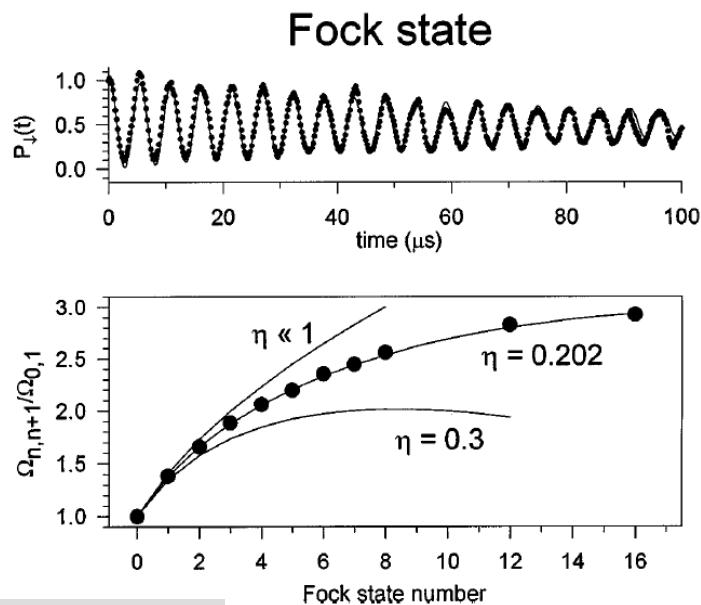
which holds for $|y| < 1$. Thus by comparing Eqs. (B3) and (B4) we obtain the expression

$$\langle m | D(\xi) | n \rangle = (n!/m!)^{1/2} \xi^{m-n} e^{-|\xi|^2/2} L_n^{(m-n)}(|\xi|^2). \quad (B5)$$

$$\langle m | D(\xi) | n \rangle = \left(\frac{n!}{m!}\right)^{1/2} \xi^{m-n} e^{-|\xi|^2/2} L_n^{(m-n)}(|\xi|^2)$$

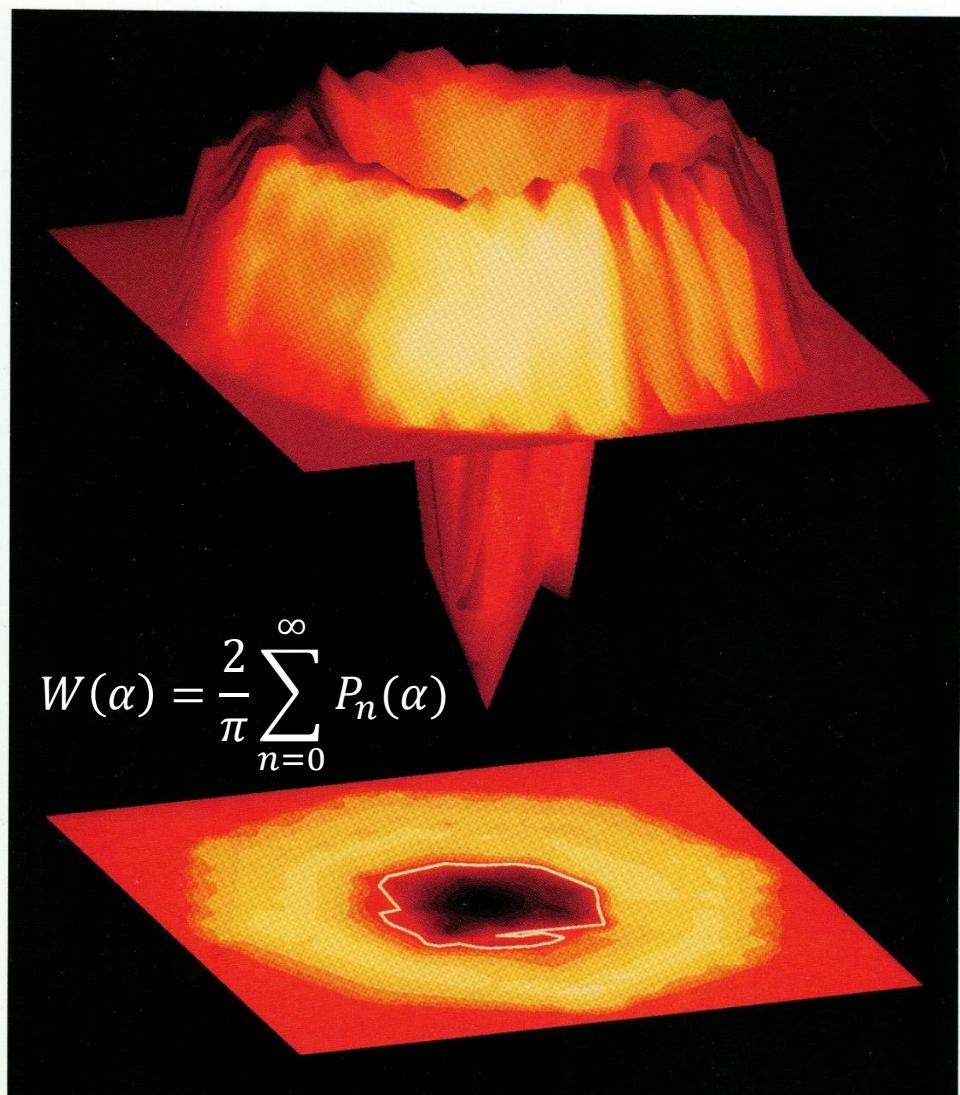
APRIL 1998

Rabi flopping on
the blue sideband



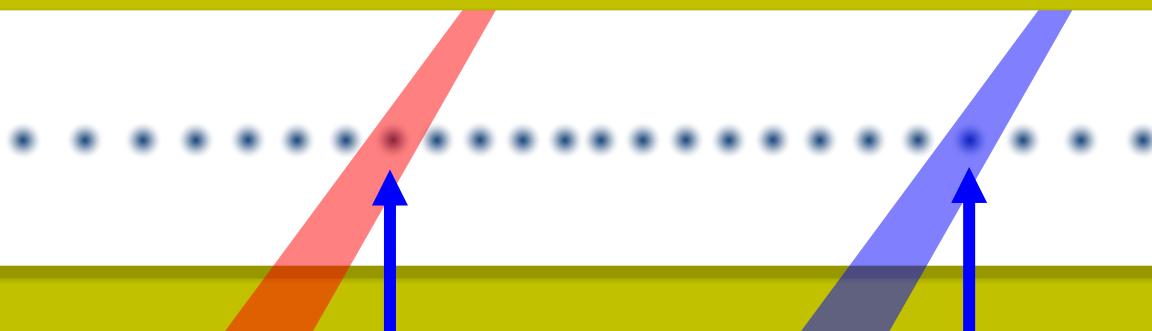
Phys. Rev. Lett. 76, 1796 (1996)

Phys. Rev. Lett. 77, 4281 (1996)



Entangling Trapped Ions

Cirac and Zoller model



Internal states of these ions entangled

Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

CM, et al., Phys. Rev. Lett. **74**, 4714 (1995)

Q. Turchette, et al., Phys. Rev. Lett. **81**, 3631 (1998)

F. Schmidt-Kaler, et al., Nature 422, 408 (2003)

A “Schrödinger Cat” Superposition State of an Atom

C. Monroe,* D. M. Meekhof, B. E. King, D. J. Wineland

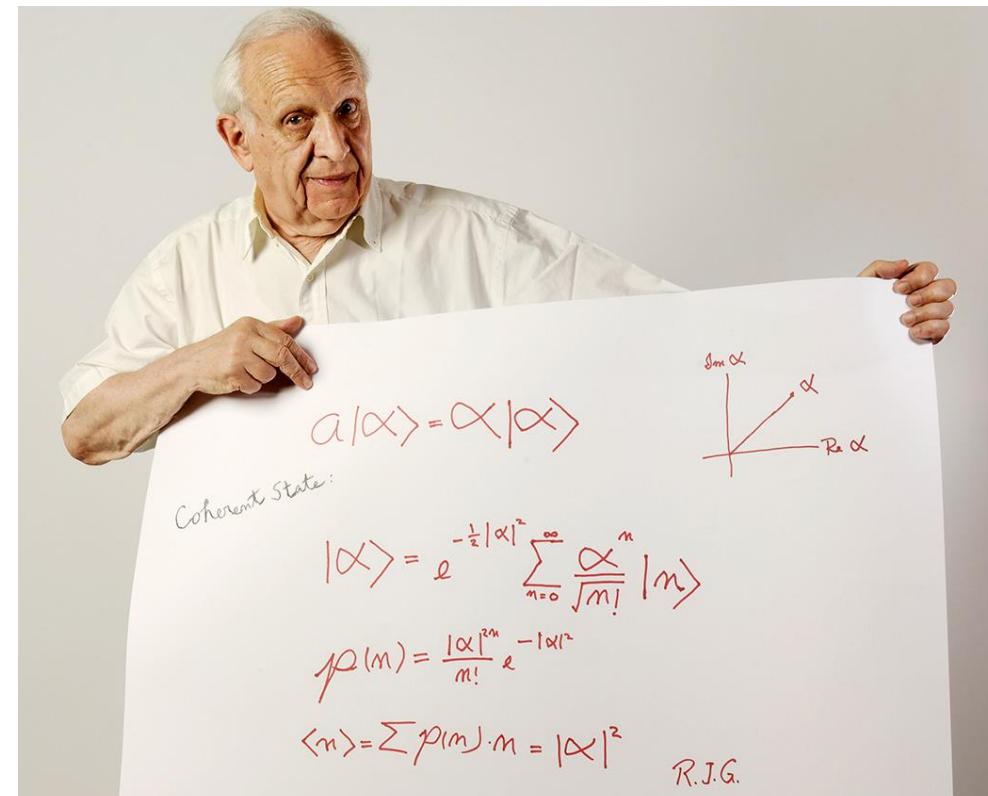
A “Schrödinger cat”-like state of matter was generated at the single atom level. A trapped ${}^9\text{Be}^+$ ion was laser-cooled to the zero-point energy and then prepared in a superposition of spatially separated coherent harmonic oscillator states. This state was created by application of a sequence of laser pulses, which entangles internal (electronic) and external (motional) states of the ion. The Schrödinger cat superposition was verified by detection of the quantum mechanical interference between the localized wave packets. This mesoscopic system may provide insight into the fuzzy boundary between the classical and quantum worlds by allowing controlled studies of quantum measurement and quantum decoherence.

Quantum mechanics allows the preparation of physical systems in superposition states, or states that are “smeared” between two or more distinct values. This curious principle of quantum mechanics (1) has been extremely successful at describing physical behavior in the microscopic

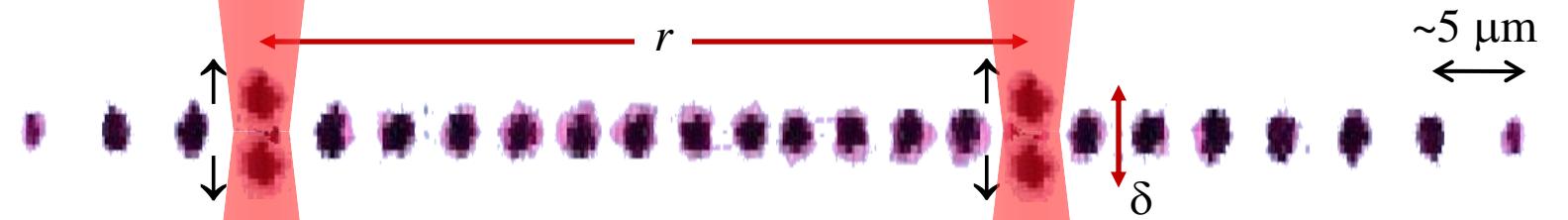
state of the system can be represented by the entangled quantum mechanical wave function,

$$\psi = \frac{| \otimes \rangle | \uparrow \rangle + | \otimes \rangle | \downarrow \rangle}{\sqrt{2}} \quad (1)$$

$$\hat{U}(t) = \hat{D}(\alpha) | \uparrow \rangle \langle \uparrow | + \hat{D}(-\alpha) | \downarrow \rangle \langle \downarrow |$$



Quantum Entanglement of Trapped Ions



dipole-dipole coupling

$$\Delta E = \frac{e^2}{\sqrt{r^2 + \delta^2}} - \frac{e^2}{r} \approx -\frac{(e\delta)^2}{2r^3}$$

$$\begin{aligned}\delta &\sim 10 \text{ nm} \\ e\delta &\sim 500 \text{ Debye}\end{aligned}$$

$$\begin{array}{ll} |\downarrow\downarrow\rangle \rightarrow & |\downarrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \rightarrow e^{-i\varphi} & |\downarrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \rightarrow e^{-i\varphi} & |\uparrow\downarrow\rangle \\ |\uparrow\uparrow\rangle \rightarrow & |\uparrow\uparrow\rangle \end{array}$$

$$\varphi = \frac{\Delta Et}{\hbar} = \frac{e^2 \delta^2 t}{2 \hbar r^3} = \frac{\pi}{2}$$

for full
entanglement

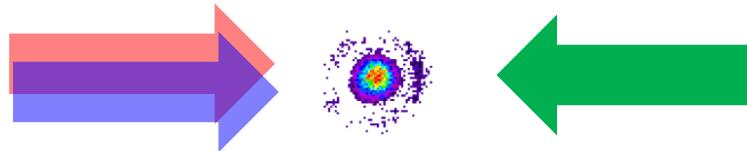
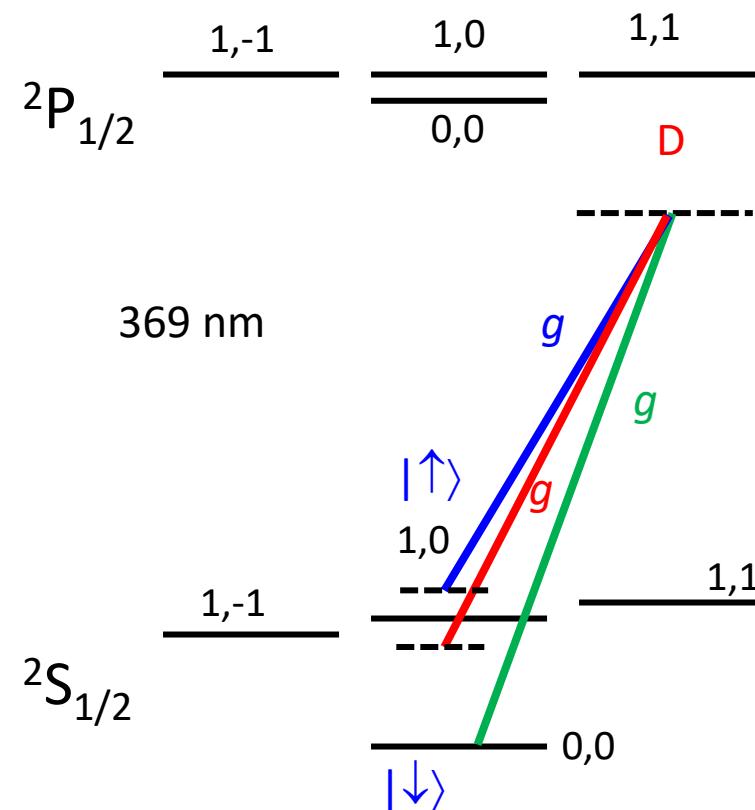
Native Ion Trap Operation: “Ising” gate

$$XX[\varphi] = e^{-i\sigma_x^{(1)}\sigma_x^{(2)}\varphi} \quad T_{\text{gate}} \sim 10-100 \mu\text{s}$$

$\text{F} \sim 98\% - 99.9\%$

Cirac and Zoller (1995)
Mølmer & Sørensen (1999)
Solano, de Matos Filho, Zagury (1999)
Milburn, Schneider, James (2000)

Spin-dependent force (single ion)



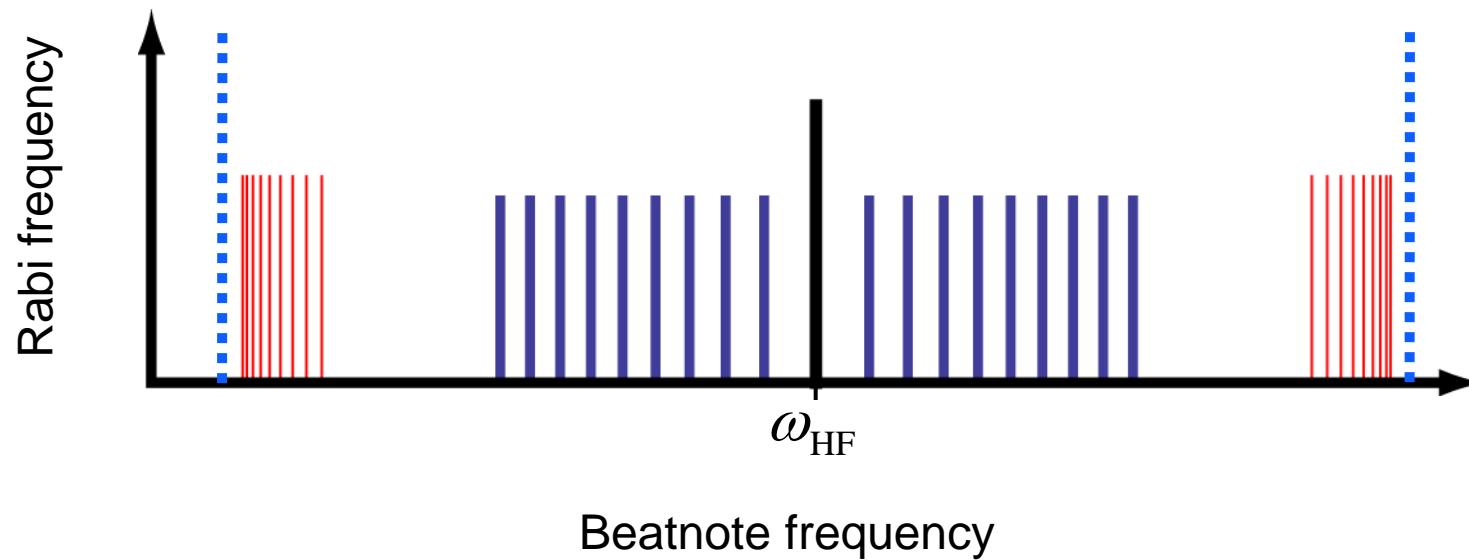
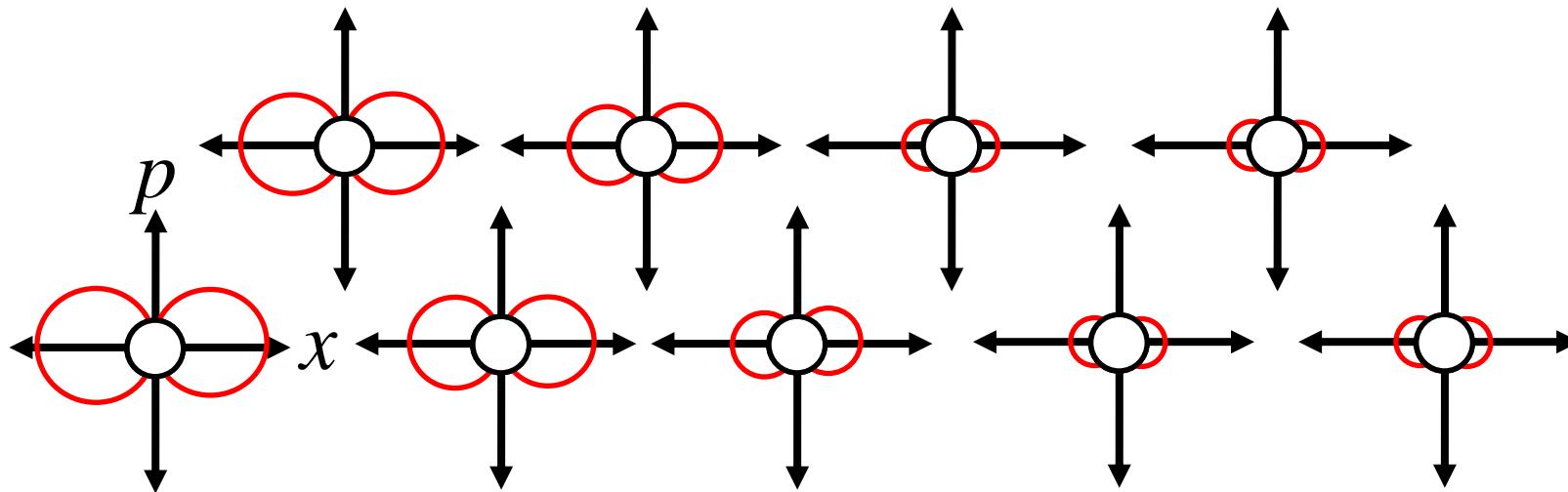
Red+blue sideband applied
simultaneously

$$\begin{aligned} H &= \eta\Omega(\sigma_+a + \sigma_-a^\dagger) \\ &\quad + \eta\Omega(\sigma_-a + \sigma_+a^\dagger) \\ &= \eta\Omega \sigma_x(a + a^\dagger) \\ &= \Omega \sigma_x(\Delta k \cdot \hat{x}) \end{aligned}$$

Lamb-Dicke
parameter

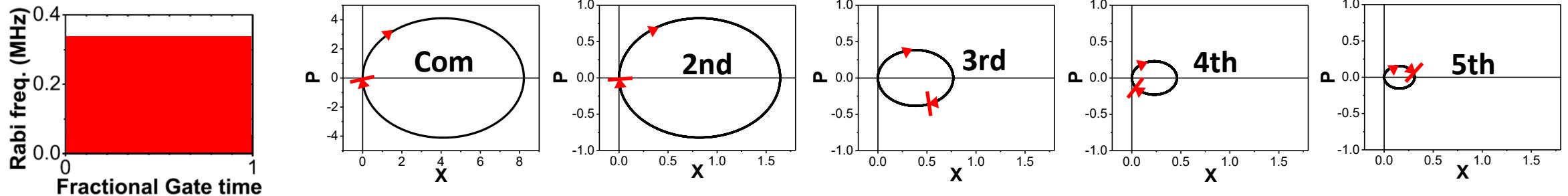
$$\begin{aligned} \eta &= \Delta k x_0 \\ \Omega &= \frac{g^2}{2\Delta} \end{aligned}$$

Phase Space evolutions

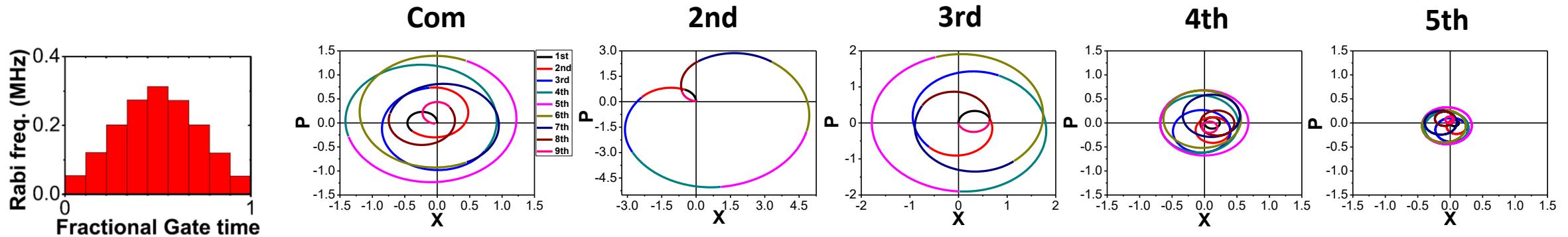


Closing Phase Space(s) N=5 ions: Addressing ions #2 & #5

constant pulse (3rd, 4th, 5th modes are not closed) tuned blue of COM



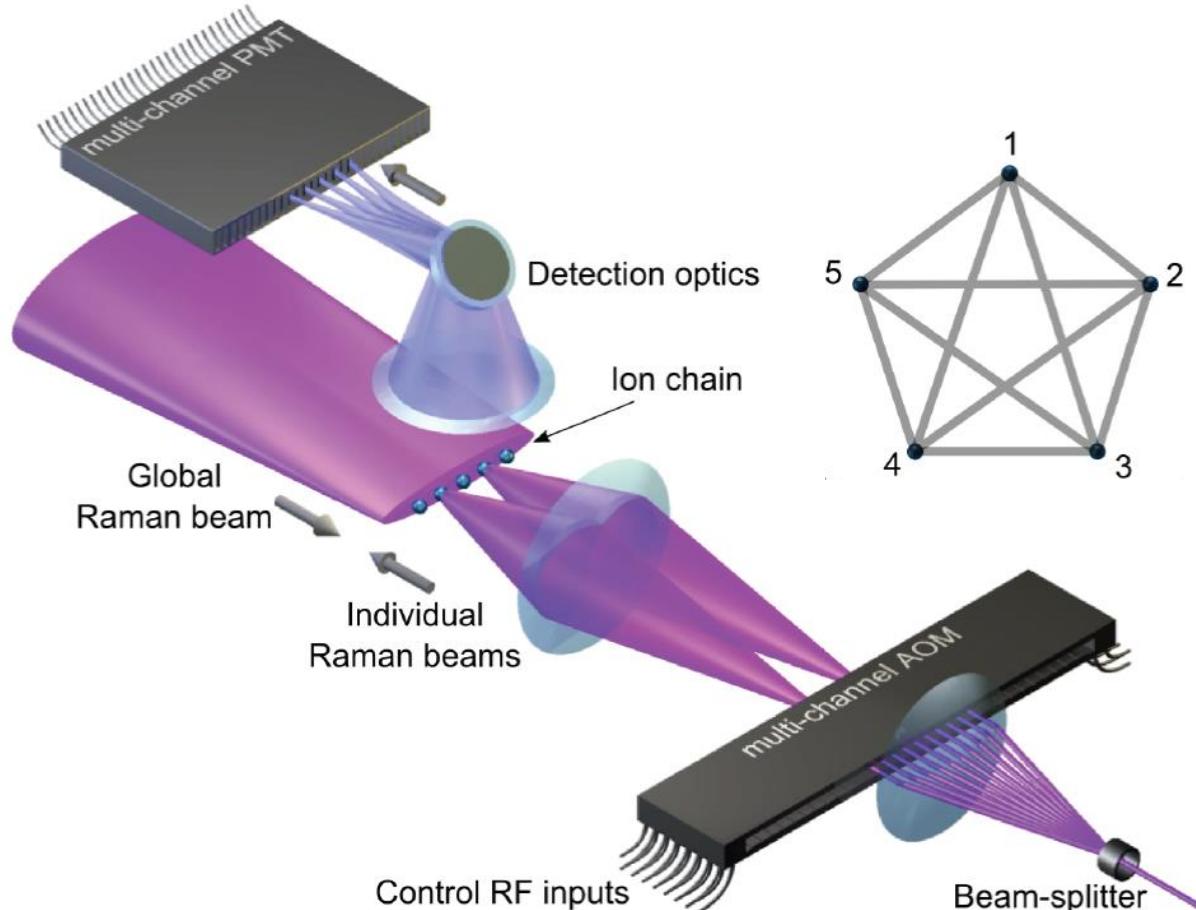
Modulated pulse (9 segments AM) (all modes are “mostly” closed)



$$XX_{2,5}[\varphi] = e^{-i\sigma_x^{(2)}\sigma_x^{(5)}}\varphi$$

- S.-L. Zhu, et al., Europhys Lett. 73 (4), 485 (2006)
 T. Choi, et al., Phys. Rev. Lett. 112, 19502 (2014)
 S. Debnath, et al., Nature 536, 63 (2016)

Programmable/Reconfigurable Quantum Computer Module

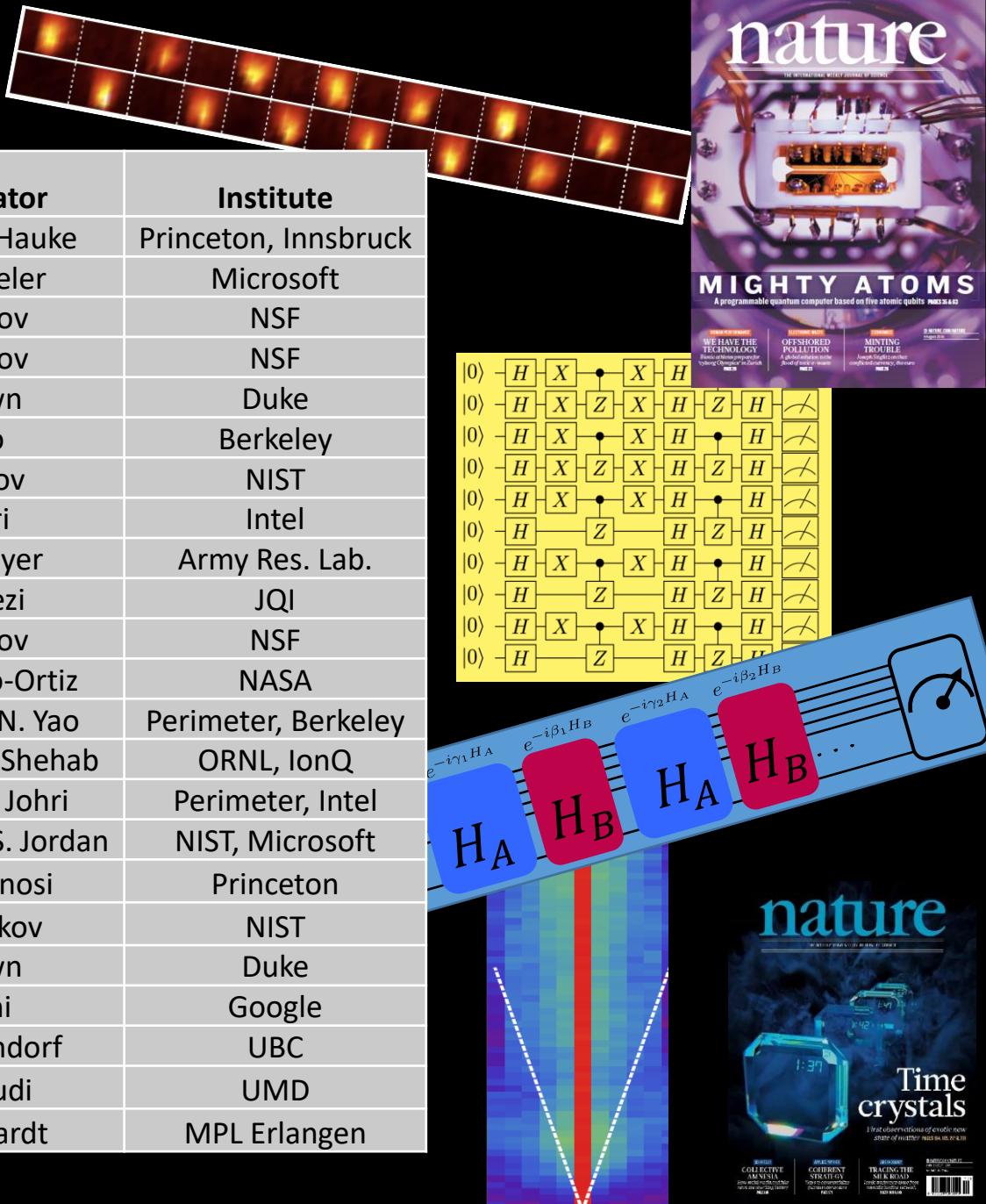


Full “Quantum Stack” architecture

User	Quantum Algorithms: <i>Deutsch-Jozsa, QFT, etc.</i>
Quantum compiler	Universal gates: <i>Hadamard, C-NOT, C-Phase, etc.</i> Native gates: <i>XX-Gates, R-gates</i>
Quantum control	Pulse shaping: <i>Optimization of XX- and R-Gates</i>
Hardware	Optical addressing: <i>Qubit manipulation/ detection</i> Ion trap: <i>Linear ion-chain, optical access, etc.</i>

S. Debnath, et al., *Nature* **536**, 63 (2016)
N. Linke, et al., *PNAS* **114**, 13 (2017)

Build it and they will come...



Application	#qubits	# gates	Reference	Collaborator	Institute
Manybody Localization	10	global	Nat. Physics 12, 907 (2016)	D. Huse, P. Hauke	Princeton, Innsbruck
Hidden Shift, Toffoli-3 Gate	5	12-60	PNAS 114, 13 (2017)	M. Roetteler	Microsoft
Grover	5	65	Nat. Comm. 8, 1918 (2017)	D. Maslov	NSF
Toffoli-4 Gate	5	33	Thesis, Debnath (2017)	D. Maslov	NSF
[[4,2,2]] Error Detection	5	27-32	Sci. Adv. 3, e1701074 (2017)	K. Brown	Duke
Time Crystal	10	global	Nature 543, 217 (2017)	N. Yao	Berkeley
Dynamical Phase Transition	53	global	Nature 551, 601 (2017)	Gorshkov	NIST
Fredkin Gate, Fermi-Hubbard	5	163	PRA 98, 052334 (2018)	S. Johri	Intel
Bayesian Game	5	20	QST 3, 045002 (2018)	N. Solmeyer	Army Res. Lab.
Qubit Detection ML	5	n/a	J. Phys. B 51 174006 (2018)	M. Hafezi	JQI
Full Adder, Parallel CNOTs	4	20	Nature 567, 61 (2019)	D. Maslov	NSF
Generative Modeling ML	4	48	Science Adv. (appear 2019)	A. Perdomo-Ortiz	NASA
Quantum Scrambling	7	45	Nature 567, 61 (2019)	B. Yoshida, N. Yao	Perimeter, Berkeley
Deuteron VQE Simulation	3	65	arXiv:1904.04338 (2019)	R. Pooser, O. Shehab	ORNL, IonQ
Circuit QAOA	9	92	arXiv:1906.02699 (2019)	T. Hsieh, S. Johri	Perimeter, Intel
Analog QAOA	15-40	global	arXiv:1906.02700 (2019)	A. Gorshkov, S. Jordan	NIST, Microsoft
Benchmarks and Comparison	7	90	arXiv: 1905.11349 (2019)	M. Martonosi	Princeton
Quasiparticle Confinement	40	global	PRL 122, 150601 (2019)	A. Gorshkov	NIST
[[9,1,3]] Bacon/Shor code	13	54	in progress (2019)	K. Brown	Duke
Sherrington-Kirkpatrick QAOA			in progress (2019)	E. Farhi	Google
Cluster state generation			in progress (2019)	R. Raussendorf	UBC
Lattice Gauge Thy			in progress (2019)	Z. Davoudi	UMD
Many-body dephasing			in progress (2019)	F. Marquardt	MPL Erlangen

Quantum Scrambling Tests

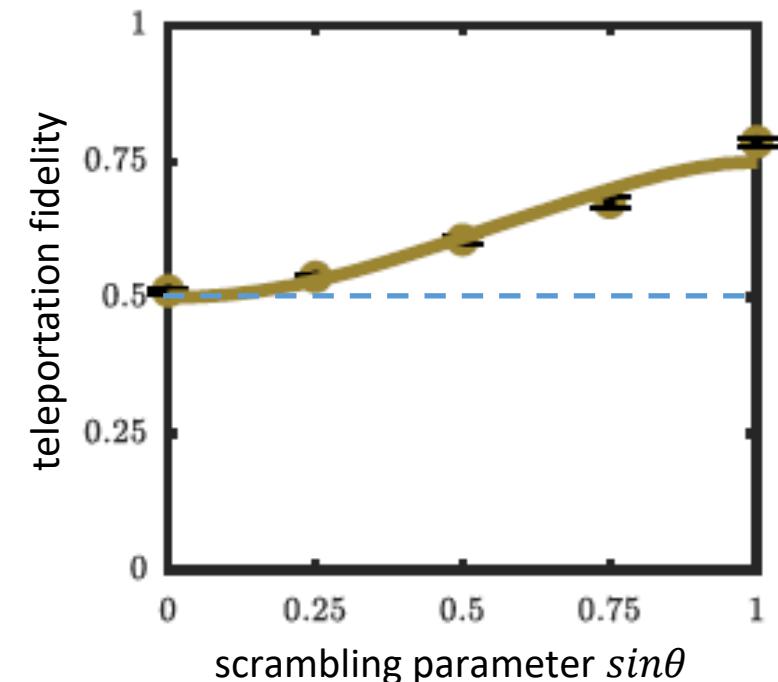
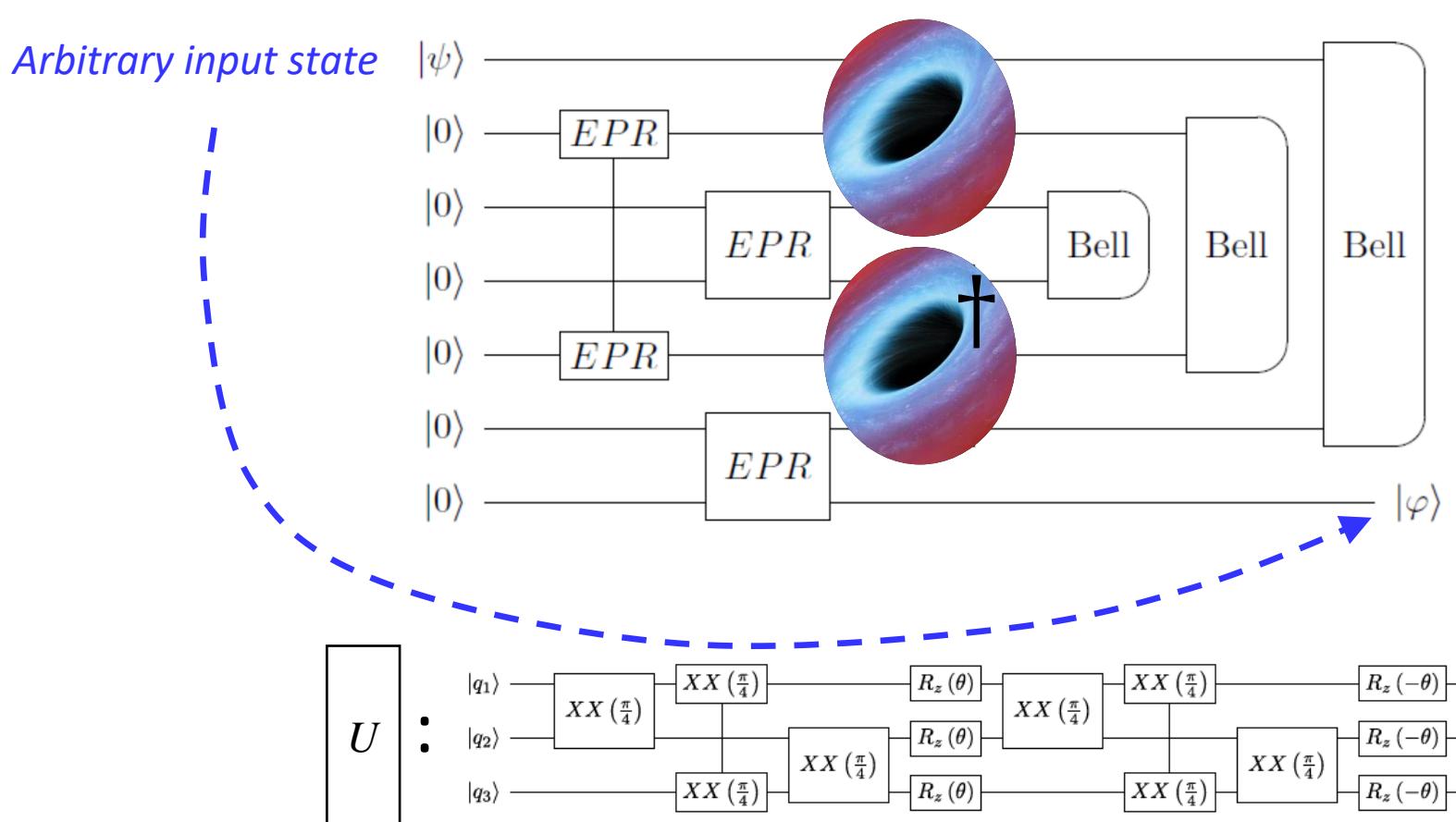
N. Yao (UC Berkeley)
B. Yoshida (Perimeter)
Nature 567, 61 (2019)

Quantum scrambling

- Not just entanglement but the “complete diffusion” of entanglement within a system
- Relevant to information evolution in black holes

P. Hayden and J. Preskill, J. HEP 9, 120 (2007)

L. Susskind and Y. Zhao, arXiv:1707.04354 (2017)



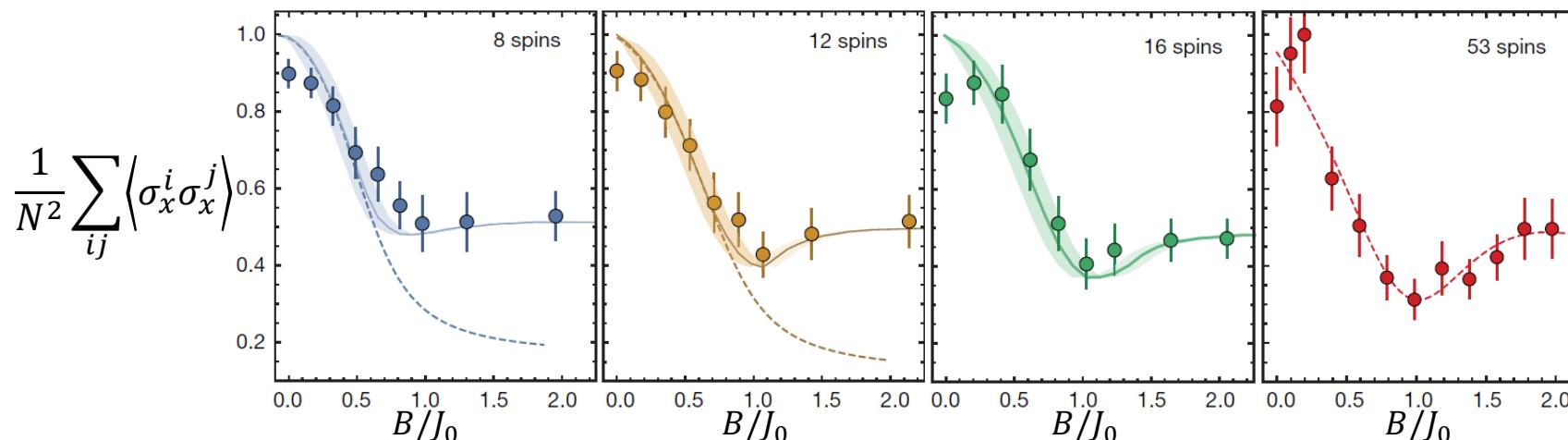
successful teleportation
if U is “scrambling”:
unambiguous litmus test
(unlike OTOC measurement)

Dynamical Phase Transition with 50+ Qubits

(1) Prepare spins along x

(2) Quench spins to $H = \sum_{i < j} \frac{J_0}{|i - j|} \sigma_x^i \sigma_x^j + B \sum_i \sigma_z^i$

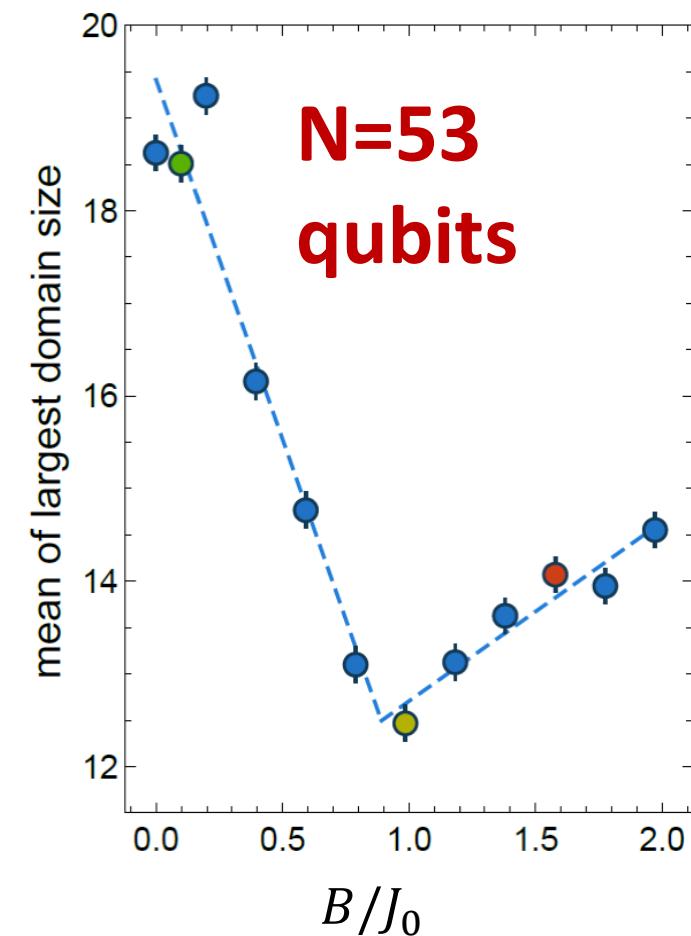
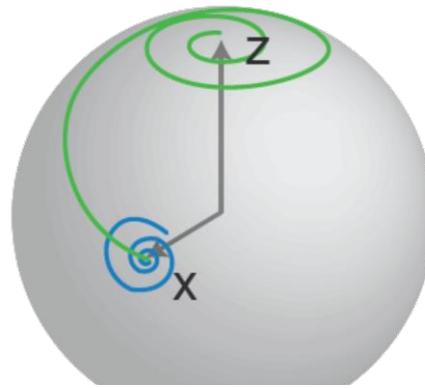
(3) Measure along x



increase
 B/J
↓



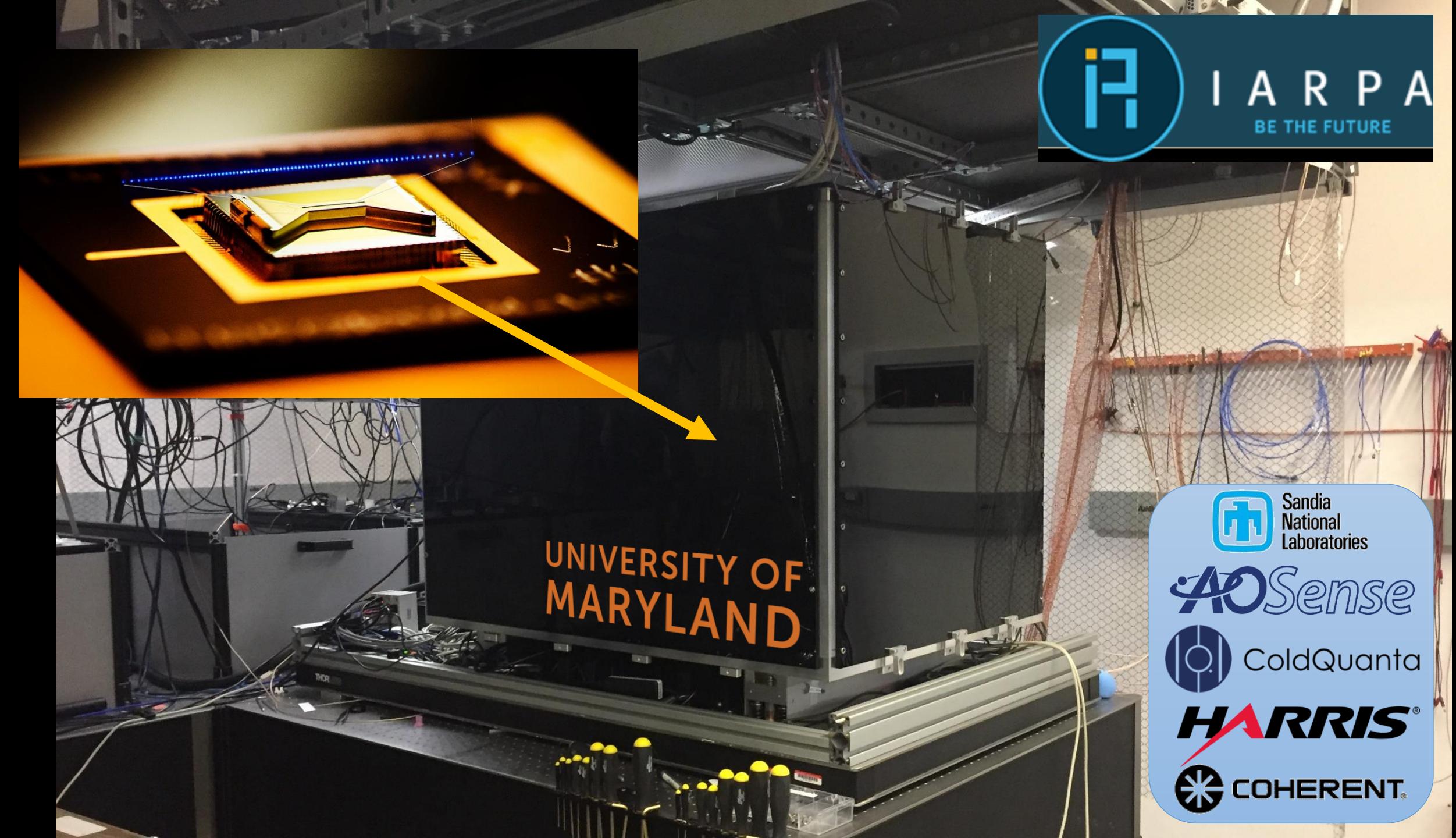
J. Zhang, et al., *Nature* **551**, 601 (2017)
see also H. Bernien, et al., *Nature* **551**, 579 (2017)





Ion Trap Lab at
JQI-Maryland

Photo: Phil Schewe



UNIVERSITY OF
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I A R P A
BE THE FUTURE



Sandia
National
Laboratories



AOSense
ColdQuanta





IONQ

System 1



System 3



System 2

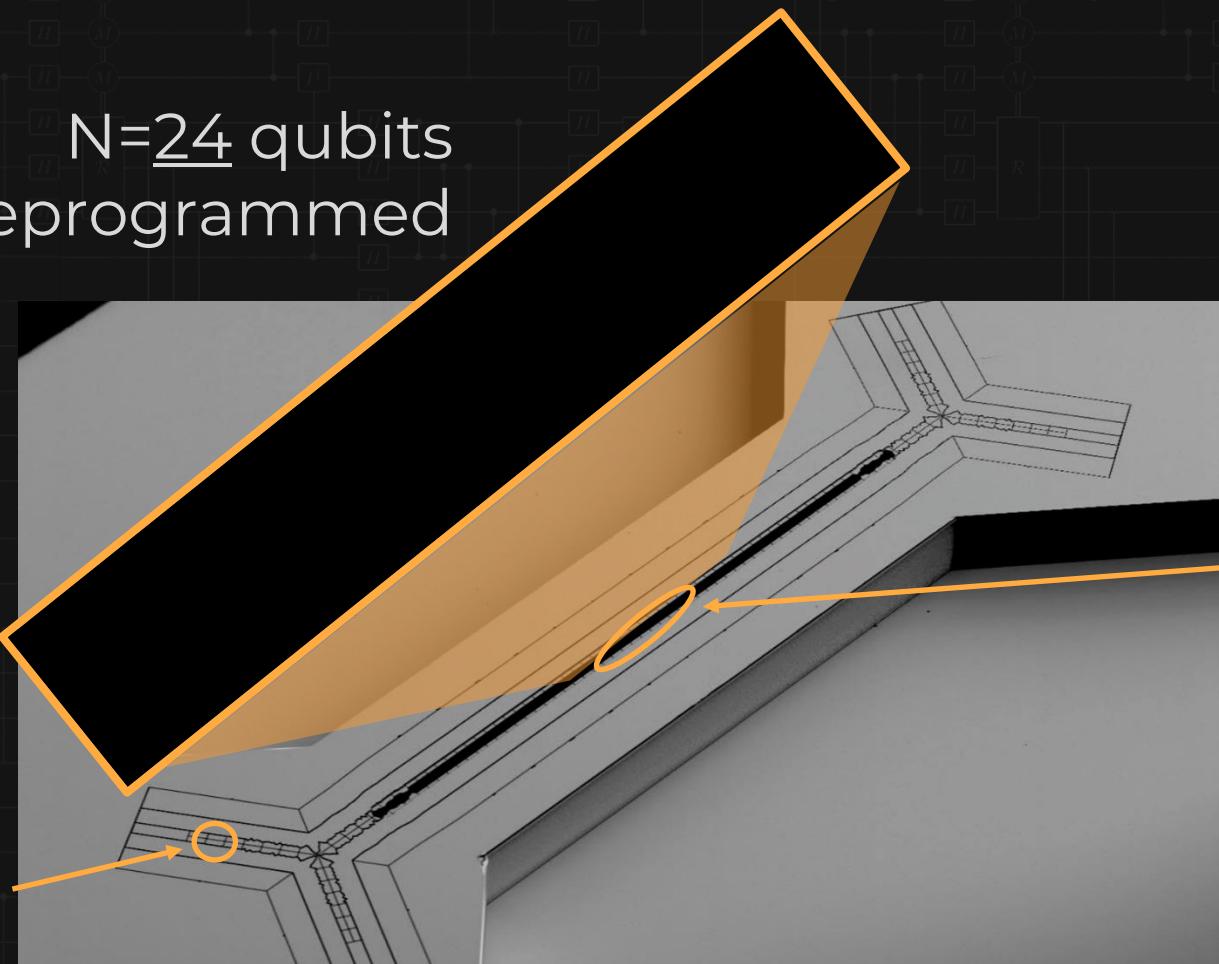


IONQ autoloading register

$N=24$ qubits
preprogrammed

Loading
zone

Quantum
Computing
zone





IONQ benchmark algorithms

Bernstein-Vazirani ‘oracle’ algorithm

Given $f(x) = \mathbf{c} \cdot \mathbf{x}$

Find n -bit string \mathbf{c}

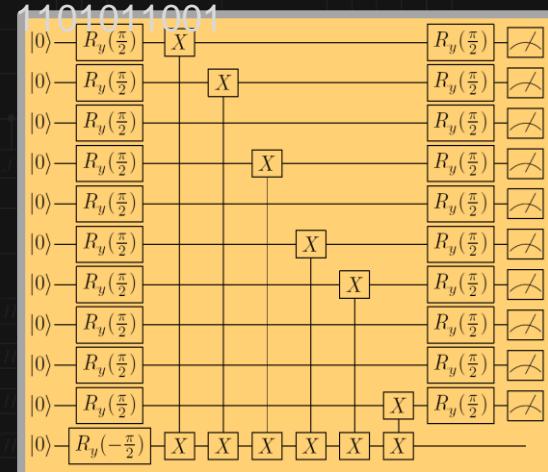
Classical
requires n queries

Quantum
requires only 1 query

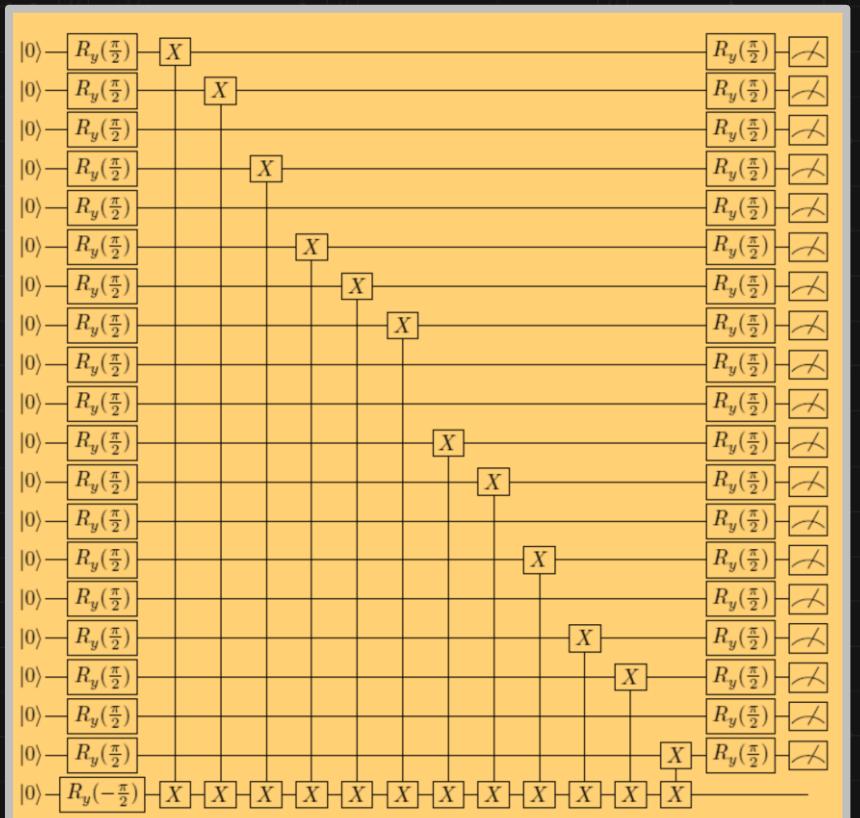
arXiv 1903.08181
(2019)

arXiv 1902.10171 (2019)

example circuit: $\mathbf{c} =$

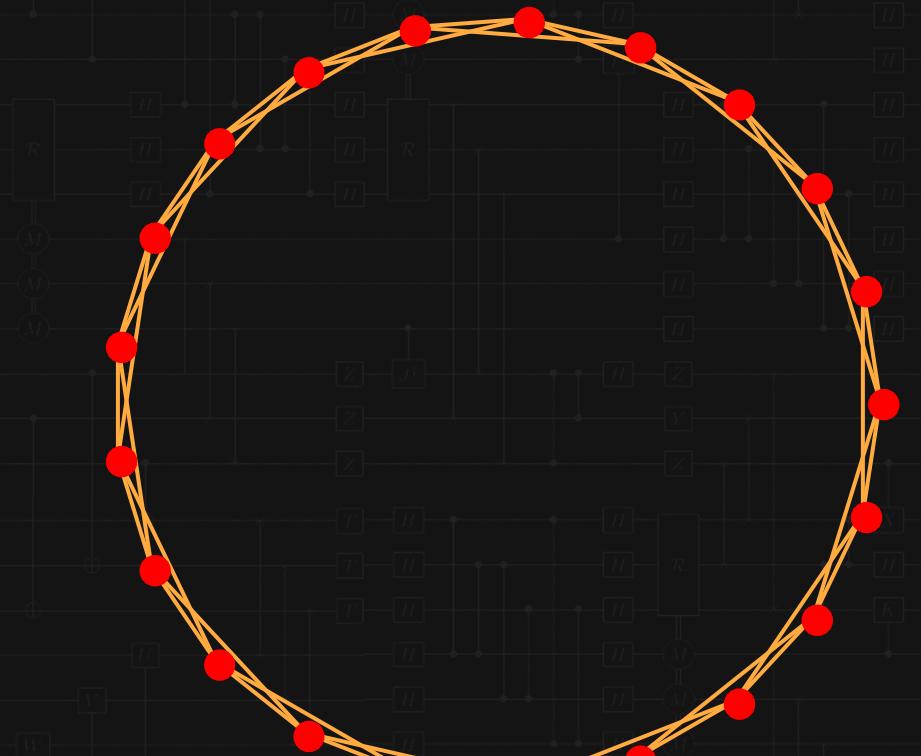
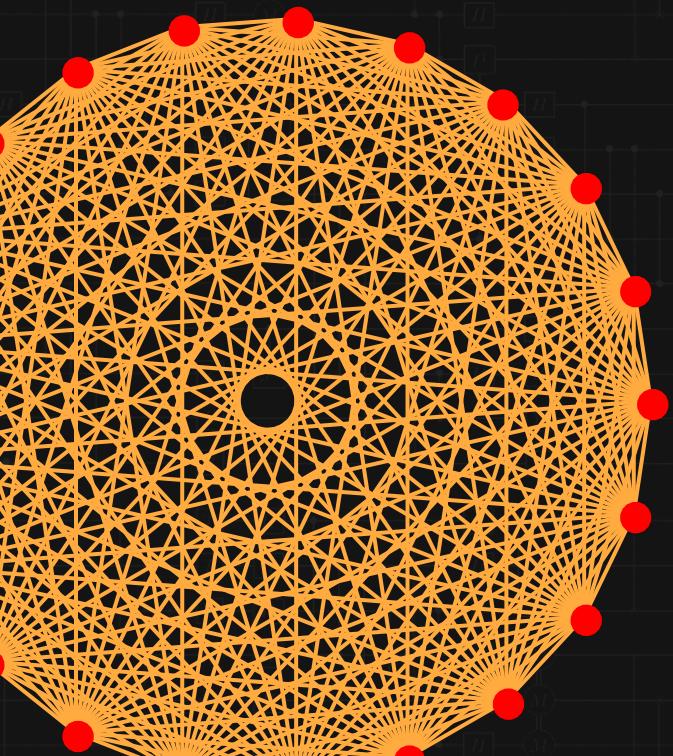


19-bit oracle: 1101011100110101101

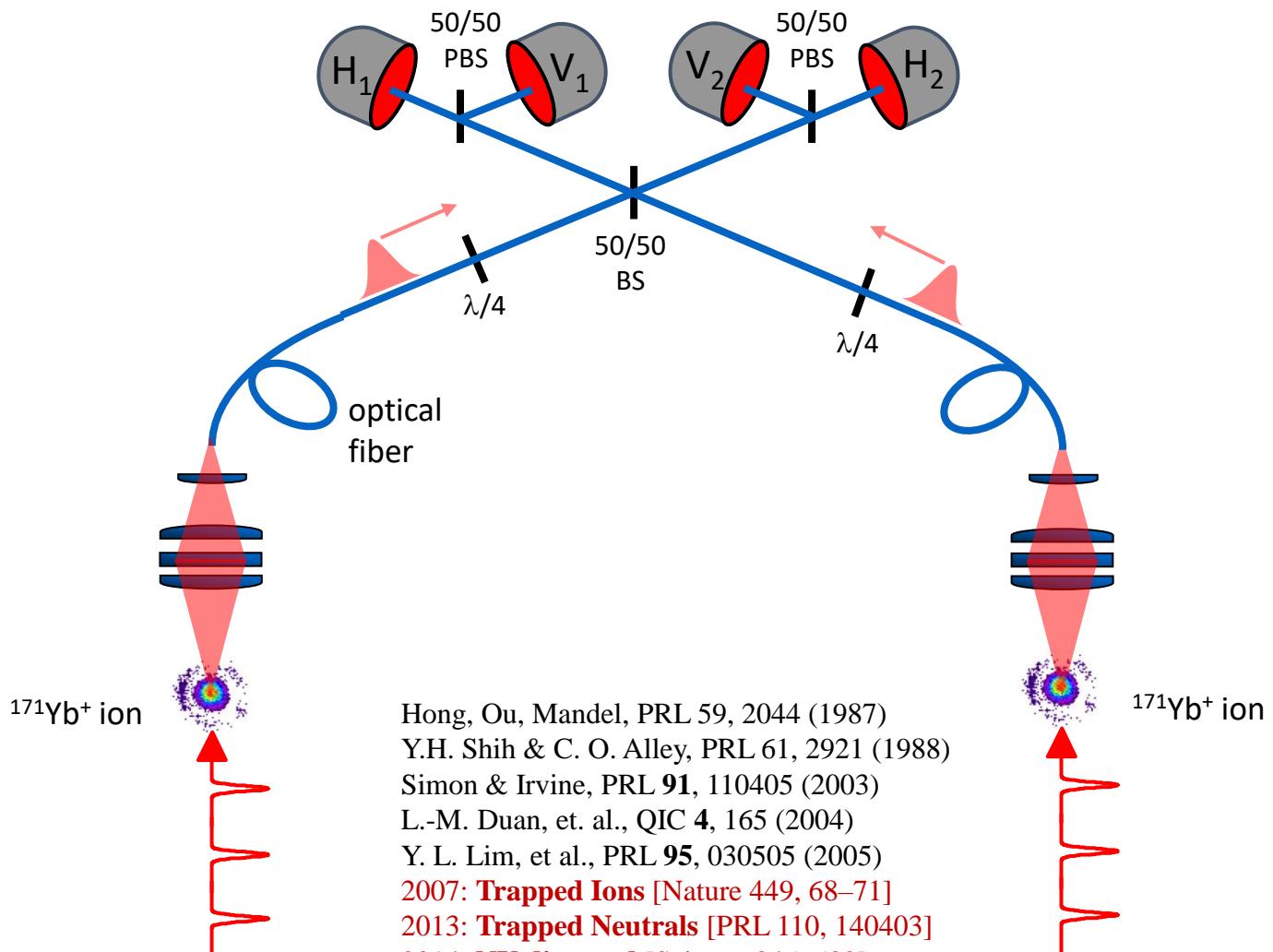


21 qubits
Fully-Connected

21 qubits
Nearest-Neighbor Connected



Scaling? Link remote emitters with photons



Heralded coincident events:

- $(H_1 \& V_2) \text{ or } (V_1 \& H_2) \rightarrow |\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle$
- $(H_1 \& V_1) \text{ or } (V_2 \& H_2) \rightarrow |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$
- $(H_1 \& H_1) \text{ or } (H_2 \& H_2) \rightarrow |\downarrow\downarrow\rangle$
- $(V_1 \& V_1) \text{ or } (V_2 \& V_2) \rightarrow |\uparrow\uparrow\rangle$

$$R_{ent} = \frac{1}{2} R p^2$$

Current State-of-art:

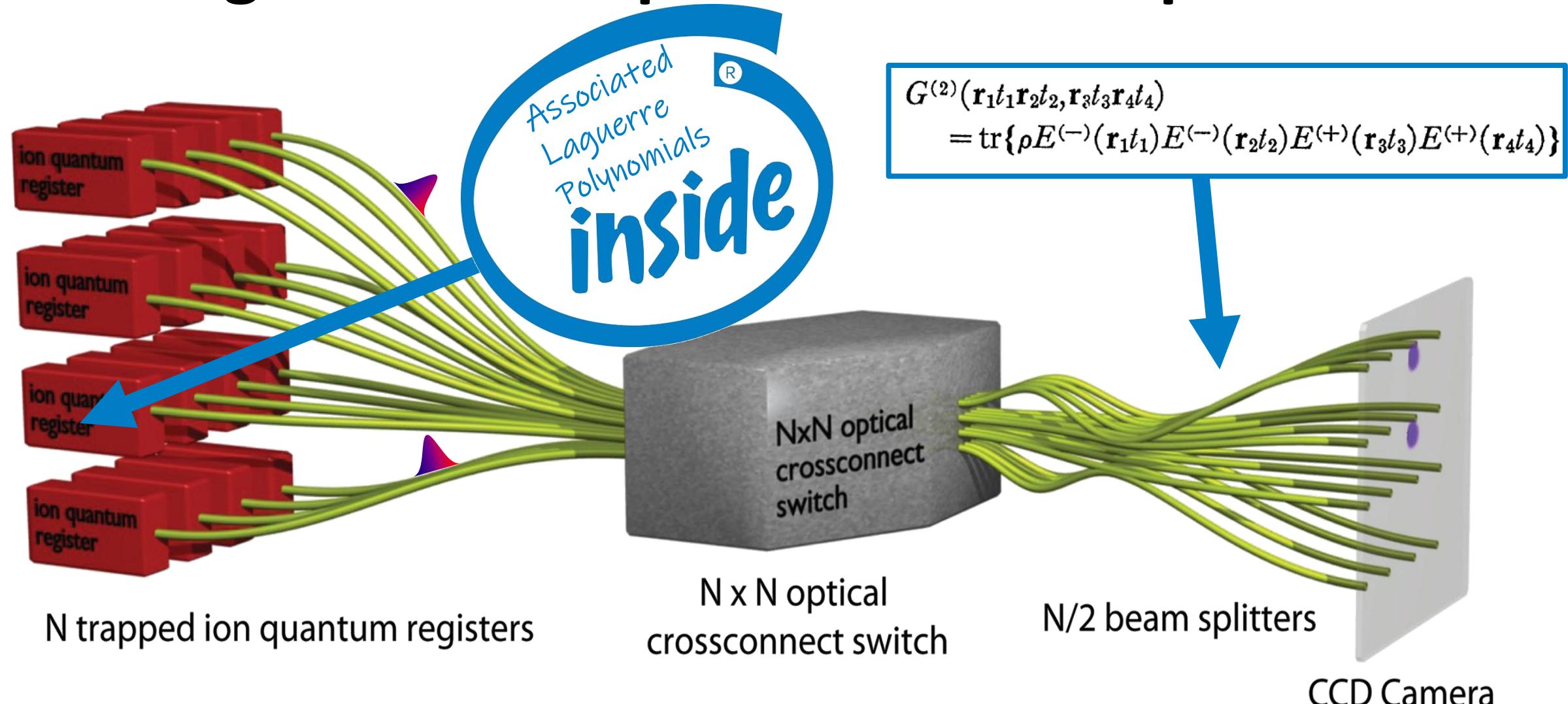
$$p = \eta_D T \frac{d\Omega}{4\pi} = (.35)(.6)(.1) = 2\%$$

$$R = 1 \text{ MHz}$$

$$R_{ent} \approx 180 \text{ sec}^{-1}$$

D. Hucul, et al., Nature Phys. 11, 37 (2015)
C. Balance, et al, arXiv:1911.10841 (2019)

Scaling The Ion Trap Quantum Computer



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Li and Benjamin, *New J. Phys.* **14**, 093008 (2012)

Monroe, et al., *Phys. Rev. A* **89**, 022317 (2014)